Difficulty of a spinning complex scalar field to be dark energy

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We comment on the possibility of a spinning complex scalar field to be dark energy. We show that it deforms (almost) completely into a nontopological soliton state, a Q ball, and the equation of state becomes that of the matter or radiation, depending on the fate of the Q ball. Thus, the spinning complex scalar field is usually very difficult to play the role of the dark energy. We also show the general condition that the spinning complex scalar field can successfully be the dark energy.

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Recent supernovae observation reveals that the expansion of the universe is accelerating [1]. In order to explain the fact, we need the cosmological constant whose fraction to the critical density is 0.7, for example. However, not only the cosmological constant plays a role for the accelerating universe, but also something which has a negative pressure, called *dark energy* in the literature.

Einstein equation tells us that $\ddot{a}/a \propto -(\rho+3p)$. Thus, when the equation of state is given by $p = w\rho$, $w < -1/3$ should hold for the dark energy to be the source of the accelerating universe. For example, the cosmological constant has $w = -1$. On the other hand, a homogeneous scalar field has $w = (T-V)/(T+V)$, where T and V denote the kinetic and potential energy of the field, respectively, and it is possible to have $-1 \leq w < -1/3$. Thus, a scalar field, called quintessence, can be a good candidate for the dark energy.

Recently it was proposed that a spinning complex scalar field with $U(1)$ potential could play a role of the dark energy, which has some different feature from quintessence, and dubbed *spintessence* [2]. (See also [3].) They show that a complex scalar field can be regard as quintessence if the field does not rotate so much ($\omega \lesssim H$, where $\omega \equiv \dot{\theta}$ is a phase velocity and H the Hubble parameter). In the opposite case, when the field rotates (or rapidly spins) in the potential ($\omega \gg H$), it will be another category of the dark energy, spintessence. Most important differences of its characters from that of the quintessence are the smallness of the changes of the equation of state and the different features in the fluctuation spectra [2]. Although they just comment on the possibility of the creation of nontopological solitons, the formation of nontopological solitons, or Q balls, is very generic for a complex field, as we have shown in the context of the Affleck-Dine baryogenesis [4–7]. In this letter, we show that a spinning complex scalar field may be difficult to be the dark energy ($w < -1/3$), and it will most likely to deform into Q balls, and the equation of motion becomes that of the matter or radiation depending on the fate of the Q ball created. We will also show the general condition of the possibility for the spinning complex scalar field to be successful dark energy.

A Q ball is a kind of nontopological soliton whose stability is guaranteed by some charge Q [8], which is a $U(1)$ charge for a complex field with $U(1)$ symmetric potential. The condition for the Q ball to exist is that $V(\Phi)/|\Phi|^2$ has the minimum at $\Phi \neq 0$. This condition is usually met for the potential of the scalar field whose curvature is negative. $w < -1/3$ leads to the potential flatter than $V \sim |\Phi|$.

For the scalar field which has negative pressure is generally unstable, and the fluctuations develop. Moreover, the curvature of the effective potential for the scalar field with $w < -1/3$ is negative, which also leads to spatial instabilities. We first show the instability band. We write a complex field as $\Phi = (\phi e^{i\theta})/\sqrt{2}$, and decompose into homogeneous parts and fluctuations: $\phi \to \phi + \delta\phi$ and $\theta \to \theta + \delta\theta$. Assuming that the gravity effects are weak, which is a good approximation here, we obtain the equation of motion of the Φ field as [9,10,4,5,7]

$$\ddot{\phi} + 3H\dot{\phi} - \dot{\theta}^2\phi + V'(\phi) = 0, \tag{1}$$

$$\phi\ddot{\theta} + 3H\phi\dot{\theta} + 2\dot{\phi}\dot{\theta} = 0, \tag{2}$$

for the homogeneous mode, and

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - 2\dot{\theta}\phi\delta\dot{\theta} - \dot{\theta}^2\delta\phi - \frac{\nabla^2}{a^2}\delta\phi + V''(\phi)\delta\phi = 0, \tag{3}$$

$$\phi\delta\ddot{\theta} + 3H\phi\delta\dot{\theta} + 2(\dot{\phi}\delta\dot{\theta} + \dot{\theta}\delta\dot{\phi}) - 2\frac{\dot{\phi}}{\phi}\dot{\theta}\delta\phi - \phi\frac{\nabla^2}{a^2}\delta\theta = 0 \tag{4}$$

for fluctuations. Equation (2) represents the conservation of the charge (or number) within the physical volume: $Q = \dot{\theta}\phi^2 a^3 = const.$

We seek for the solutions in the form

$$\delta\phi = \delta\phi_0 e^{\alpha(t)+ikx}, \qquad \delta\theta = \delta\theta_0 e^{\alpha(t)+ikx}. \tag{5}$$

If α is real and positive, these fluctuations grow exponentially, and go nonlinear to form Q balls. Inserting these forms into Eqs.(3), we get the following condition for nontrivial $\delta\phi_0$ and $\delta\theta_0$,

$$\left(\ddot{\alpha} + \dot{\alpha}^2 + 3H\dot{\alpha} + \frac{k^2}{a^2} + V'' - \dot{\theta}^2\right) \times \left(\ddot{\alpha} + \dot{\alpha}^2 + 3H\dot{\alpha} + \frac{k^2}{a^2} + \frac{2\dot{\phi}\dot{\alpha}}{\phi}\right) + 4\dot{\theta}^2\dot{\alpha}^2 = 0. \tag{6}$$

This equation can be simplified to be

$$\dot{\alpha}^4 + \left(2\frac{k^2}{a^2} + V'' + 3\dot{\theta}^2\right)\dot{\alpha}^2 + \left(\frac{k^2}{a^2} + V'' - \dot{\theta}^2\right)\frac{k^2}{a^2} = 0, \tag{7}$$

where we assume that cosmological expansion is negligible, $H \sim 0$, so that the orbit of the field in the potential is circular: $\phi \sim const$. We also assume that $\ddot{\alpha} \ll \dot{\alpha}^2$.

In order for α to be real and positive, we must have the last term of Eq.(7) to be negative, so that the instability band for the fluctuations is

$$0 < \frac{k^2}{a^2} < \dot{\theta}^2 - V''. \tag{8}$$

Notice that the instability band always exist, since the curvature of the potential is negative for $w < -1/3$.

Q balls are produced with the typical size $\sim k_{res}^{-1}$, where k_{res} denote the most amplified mode in the instability band. This process generally takes place nonadiabatically as we showed in Refs. [4,5,7]. Once the Q balls are produced, they act like a (dark) matter, so the energy density evolves as a^{-3}. We also show that (almost) all the charges of the field are absorbed into the produced Q balls, so that there is no homogeneous field left to be a dark energy. For the later fate of the Q ball, the most important feature of the Q ball is its mass in the function of the charge. It can be written as

$$M_Q \sim m_{\phi,eff} Q^p, \tag{9}$$

where $m_{\phi,eff} \sim |V''|$ at the Q-ball formation time, and $3/4 \leq p \leq 1$, depending on the shape of the potential. For example, $p = 3/4$ for the flat potential [11]. The Q ball is stable against the decay into some other particles, if the mass per unit charge is smaller than the mass of those particles:

$$\frac{M_Q}{Q} \sim m_{\phi,eff} Q^{-(1-p)} < \frac{m_{decay}}{q}, \tag{10}$$

where m_{decay} is the smallest mass of the particles which has the same $U(1)$ charge as the complex field Φ, and q is the charge carried by this particle. If this condition holds, Q balls with large enough charge to be stable against the decay into other particles can exist in the present universe, and it can be a dark matter if their energy density has a crucial fraction to the critical density. For the smaller charge, Q balls disappear through the charge evaporation from the Q-ball surface, and Φ-particles may becomes a component of the radiation. On the other hand, they decay into (lighter) particles, and those particles contribute to the radiation, and may be some fraction of the (dark) matter depending on their masses. Therefore, the energy density evolves as a^{-4} or a^{-3}. In any case, they cannot contribute to the dark energy, whose energy density evolves as a^{-r} with $r < 3$.

One may wonder how general the Q ball formation occurs for the spinning complex scalar field dark energy. We will seek for the possibility of the potential which does not lead to Q-ball formation, while the field still acts as the dark energy. † The kinetic energy is written as

$$|\dot{\Phi}|^2 = \frac{1}{2}(\dot{\phi}^2 + \dot{\theta}^2\phi^2) \simeq \frac{1}{2}\dot{\theta}^2\phi^2, \tag{11}$$

where we assume $\dot{\theta} \gg H$ and $\dot{\phi} \approx 0$ in the last equality. In order for the accelerating expansion, we need $w < -1/3$, so

$$\dot{\theta}^2 < \frac{V(\phi)}{\phi^2}, \tag{12}$$

where we use $V(\Phi) = V(\phi)$ because of the $U(1)$-symmetric potential. From Eq.(1) with $H \approx 0$, we have $\phi\dot{\theta}^2 \simeq V'(\phi)$, so that

$$\ddot{a} > 0 \quad \rightarrow \quad \phi V'(\phi) < V(\phi). \tag{13}$$

Rewriting this condition in terms of $f(\phi) \equiv V(\phi)/\phi^2$, we obtain

$$f(\phi) + \phi f'(\phi) < 0. \tag{14}$$

On the other hand, the condition of the existence of the Q ball is that the function $f(\phi)$ has the minimum value f_{min} at nonzero ϕ:

$$f = f_{min} \quad \text{at} \quad \phi = \phi_* \neq 0. \tag{15}$$

Suppose the situation that the Q-ball condition (15) does not hold. We will find whether the dark-energy condition (13), or, equivalently, (14), can be satisfied in this case. The function $f(\phi)$ which violates the Q-ball condition can be divided into two types: (a) monotonously increasing function, and (b) the function which has an extremum (or some extrema), but the minimum of the function is achieved at $\phi = 0$. For the first type, $f' > 0$ is always satisfied, so that the dark-energy condition (14) cannot hold. This implies that the spinning complex scalar field in the type (a) potential cannot be the dark energy of the universe.

For example of this type, we consider the superposition of power-law potential, $V(\phi) = \sum_i A_i \phi^{q_i}$. ($q_i$'s cannot be all negative, since the condition for the complex field to spin rapidly in the potential is $(\phi^3 V'(\phi))' > 0$ [2], assuming $V(0) = 0$.) The dark-energy condition is expressed as

$$\sum_i (q_i - 1) A_i \phi^{q_i} < 0, \tag{16}$$

which can be satisfied if some i exist such that $q_i < 1$. On the other hand, $f(\phi) = \sum_i A_i \phi^{q_i - 2}$. It is easily seen that

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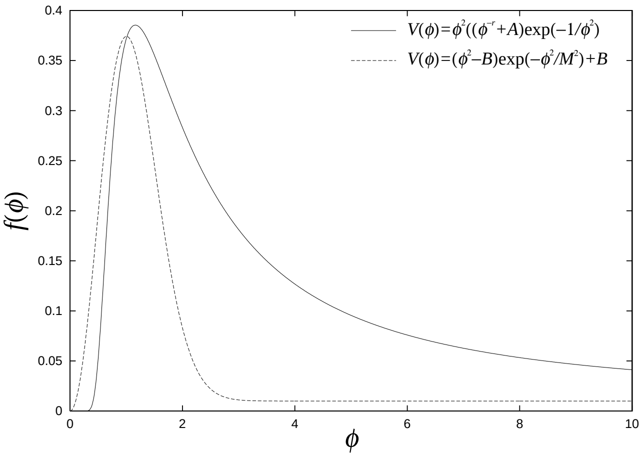

FIG. 1. Examples of $f(\phi)$ for the successful dark energy spinning complex scalar field.

$f = f_{min}$ is achieved at $\phi = \infty$ in the case that $q_i < 2$ for all i. If some j exist such that $q_j > 2$, $f'(\phi) = \sum_i (q_i - 2)A_i \phi_{q_i - 3} = 0$ is satisfied at ϕ_*, which is determined by

$$\sum_j (q_j - 2) A_j \phi_*^{q_j - 3} = \sum_k (2 - q_k) A_k \phi_*^{q_k - 3}, \qquad (17)$$

where $q_j > 2$, while $q_k < 2$. In any case, the minimum of the function $f(\phi)$ is achieved at nonzero ϕ, which leads to the Q-ball formation.

In the second category (b), it is possible for the field to be dark energy in general in some range of the field amplitude. During the field to be the dark energy, where the field feels negative pressure, fluctuations develops very effectively, and the amplitude of the field may decreases more rapidly. This may lessen the effects for the dark energy. The examples of this type of the effective potential are $V(\Phi) = |\Phi|^2((|\Phi|^2)^{-r/2} + A)\exp(-1/|\Phi|^2)$, where $1 < r < 2$ and $A > 0$, or $V(\Phi) = (|\Phi|^2 - B)\exp(-|\Phi|^2/M^2) + B$, where $B > 0$ and M is some mass scale. We show plot $f(\phi)$ of these examples in Fig.1. It seem somewhat difficult to obtain such form of the potentials, but not impossible.

Finally, we must mention the fate of the complex scalar field which is just oscillating along the radial direction in its potential. This is the very situation for the so-called oscillating inflation [12]. Even in this case, the Q-ball formation takes place naturally: both Q and anti-Q balls with the opposite sign of charges of the same order of magnitude are created [4,5,7]. Therefore, the universe cannot expand in the accelerated rate because of the same reason mentioned above.

In summary, we have shown that a spinning complex scalar field feels spatial instabilities, and deforms into Q balls very generally. Once the Q balls are produced, it acts like a (dark) matter, and the equation of state becomes $p = 0$. The most important fact is that (almost) all the charges of the complex field are absorbed into the produced Q balls, so that there is no (homogeneous) field left to be a dark energy. Concerned with the later fate of the Q balls, it depends on the shape of the potential, and they can be stable to be the dark matter, or decay into other particles to be the radiation. (Complete evaporation of Q balls also leads to the radiation component.) In either case, the energy of the complex field are altered into the form of matter or radiation, and it cannot play a role for the source of the accelerating universe, the dark energy. Of course, if the rotation speed of the field is very slow ($\omega \ll H$), the field slowly rolls down to the minimum of the potential, and acts as the quintessence, as mentioned in Ref. [2], and can be the dark energy.

We have also shown that there is some possibility for the spinning complex field to be the dark energy. It may realized if the potential of the field has somewhat unusual forms such that $f(\phi)$ should have an extremum at nonzero ϕ and its minimum at $\phi = 0$.

Concerned with symmetries other than global $U(1)$, it is known that the Q-ball type soliton exists in many cases, such as for nonabelian symmetries [13], gauged $U(1)$ [14], etc. Therefore, if the fluctuations of the field develop enough, these Q balls might be created in very similar manner, and their energy density evolves as the matters.

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